BASIC SHOCK AND VIBRATION THEORY AND ANALYSIS TECHNIQUES

VIBRATION

1. There are two basic classes of vibration problems. These are commonly classified as force excitation and motion excitation.

   Force excitation involves protecting a supporting structure from the forces generated by the supported object. An example of this is the use of motor mounts between your car's engine and the frame. The mounts allow relative motion between the engine and frame and prevent most of the engine- generated forces from being transmitted to the frame.

   The second class of vibration problems, motion excitation, is basically the reverse of force excitation. Motion excitation involves protecting a supported mass. An example of this is the padded seat in your car. The supported mass, your body, is protected from the input coming through the supporting mass, the car body. Again, this is done by allowing relative motion between the mass to be isolated and the disturbing mass.

2. What does a resilient mounting actually do? First, consider the case of rigid attachment. In this case, all forces generated in one mass are directly transmitted to the second mass. Now, consider the case of a completely free system, where there is no attachment at all between two masses. In this case, none of the forces generated in one mass will be transmitted to the other mass.

   These two cases provide the limiting conditions for all vibration isolation problems. Since it is not possible to have a completely free system, we compromise by separating the two masses with a resilient mounting.

   A resilient mounting results in less forces being transmitted, and in some cases, we can approach the completely free system.

3. What is meant by **natural frequency**, and how is it determined? The natural frequency of a system can be defined as that frequency where a spring-mass system would experience free vibration if it were disturbed. Everything has a natural frequency, i.e., a tuning fork, a guitar string.

   The natural frequency of a system can be considered a function of two quantities, mass and stiffness. There are exceptions to this (pendulum), but for all practical purposes, we will say natural frequency is only a function of mass and stiffness. Another term for stiffness is spring rate, expressed in lbs/inch.

   To determine natural frequency, it is best to consider a single degree of freedom system. That is a system that is restrained from motion in all axes except one.

$$\text{Natural frequency} = f_n = \frac{1}{2\pi} = \sqrt{\frac{k}{M}} = 3.13\sqrt{\frac{k}{W}}$$

Where K = spring rate in lbs./inch
W = weight in lbs.
and M - Mass in lb-sec2in.

From this formula, you can see that an increase in stiffness (K) or a decrease in weight (W) will result in an increase in natural frequency (f_n). Conversely, a decrease in stiffness (K) or an increase in weight (W) will result in a decrease of natural frequency.

Since the weight is usually fixed in a vibration isolation problem, the natural frequency is controlled by varying the spring rate.

4. **Vibration isolation** is obtained by mounting the unit to be isolated upon resilient mounts in such a manner that the natural frequency of the unit upon the mount is substantially less than the frequency of the vibratory excitation to be isolated. In the general case, however a discreet frequency to be isolated is not known, but a specified vibratory input per some military specification is given. When this is the case, the correct isolator natural frequency must be determined by the end use of the item, whether it is a mobile, shipboard, aircraft, helicopter, or whatever installation.

   Isolation of vibration begins at the point where the ratio of the disturbing frequency to the isolator natural frequency is equal to the square root of two or where

$$F/F_n = \sqrt{2}$$

   If isolation is required by a certain frequency, this formula will place the natural frequency at the proper point.

5. What is **transmissibility**? Transmissibility can be defined as the ratio of the output excitation to the input excitation.

   or T = Output / Input

One can see that when T is greater than one, we are not isolating but amplifying. This illustrates the importance of proper natural frequency placement. At the natural frequency of a system, the transmissibility will always be greater than one. Thus to obtain isolation, there must be some frequency of the system.

6. **Damping** in a spring-mass system -- Damping is defined as a means of dissipating energy in a spring-mass system.

   An important facet of shock or vibration isolation lies in just what an isolator does. A vibration/shock mount does not absorb energy; there is no such thing as a vibration or shock absorber. By the basic laws of physics, energy can neither be created nor destroyed within a system. Since the total energy remains constant then, how does an isolator work? A vibration isolator simply dissipates the energy from excitation by converting it to heat energy. The means for this conversion is damping.

   There are basically three types of damping: the first is viscous or fluid damping. An example of this is the shocks on your car where a piston is forced through a fluid. The resistance of the fluid on the motion of the piston causes some of the motion energy to be converted to heat and dissipated.

   The second type of damping is coulomb or friction damping. Coulomb damping is characterized by sliding surfaces. Again, motion energy is converted to heat energy and dissipated.

   There exists a third type of damping besides the two mentioned. One of these may be termed hysteresis damping. The amount of damping in an elastomer is mostly dependent on the extent to which it is filled. Elastomers may be tailored to provide certain damping characteristics within reasonable limits.

   The most commonly encountered types of damping are coulomb and hysteresis damping. Viscous damping is difficult to obtain in a usable-sized package.

   Previously, we discussed the fact that the natural frequency of a system is a function of only the mass and the spring rate. For the purposes under consideration here, there is no relationship between damping and natural frequency.

   The natural frequency of a system determines where the maximum response (resonance) occurs. The amount of damping in a system determines the magnitude of that response.

NATURE AND ISOLATION OF SHOCK

Shock is a difficult subject to discuss because the term shock has no definite and accepted meaning. Vibration on the other hand is readily definable and has been presented previously as a continuing condition in which an oscillatory force or motion exists in a repetitive manner. We can attempt to define shock in terms of what is not vibration. Shock, as contrasted to vibration, is a transient condition, where the equilibrium of a system is disrupted by a suddenly applied force or by a sudden change in the direction or magnitude of velocity. This disruption, and the ensuing reaction of the system in restoring equilibrium, constitute a condition of shock.

Isolation of vibration and shock signifies the temporary storage of energy, and its subsequent release substantially in its entirety but in a different time relation. Isolation is thus distinct from the absorption or dissipation of energy. Some dissipation of energy occurs, but this is a secondary consideration in the isolation function.

How is a shock condition imparted to an equipment? The typical shock isolation problem generally involves a specified shock input in the form of a shaped pulse, a free drop or a velocity change. These "pure' inputs are intended to simulate such environments as aircraft landing shock, handling shock, shipping environments, near miss ballistic impacts, and railroad inputs (switching, coupling, humping, slack take-up).

Typical shaped input pulses take the form of a one-half sine or sawtooth. Acceleration levels and pulse durations vary considerably with the particular application. The are intended to simulate aircraft landing shock or even a crash condition.

Free fall drops are primarily used to simulate handling and shipping conditions. For instance, an item falling off a forklift, or being dropped from a truck bed experiences a free fall drop. The free fall drop takes many forms: flat drop, rotational edge drop, corner drop, side drop, etc. The shock input associated with a free fall drop is a velocity shock. A velocity shock is defined as an abrupt change in velocity, either from zero to a finite value or from one magnitude to another or from a finite value to zero.

Velocity shock inputs are also the source of excitation for such requirements as the Navy ballistic test and railroad humping tests. The Navy has rather unique requirements in simulating shock environments. They specify and define a test machine and method rather than some fixed value of velocity change. The Navy specification that defines this test is MIL-S-901C. Equipment is divided into three classes: lightweight, medium weight tests use large hammer type machines. The heavyweight test at present is conducted on a barge around which, at defined distances and depths, explosive charges are set off.

The velocity shock associated with a railroad environment is often simulated by such tests as the inclined plate test, wherein an article is allowed to slide down an inclined plane and impact against a rigid surface. The travel distance and angle of inclination are specified.

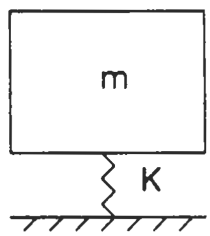
ISOLATION OF SHOCK

As indicated previously, shock is an impulsively applied force. A shock isolator is a device that isolates shock by sorting energy at a high application rate and then releasing the energy at a lower rate. The purpose of a shock isolator should be obvious to the casual observer; it is simply to reduce the shock transmission to the equipment when such an environment as described above is encountered. Because shock isolators must store energy to function, they usually require larger deflections than those associated with vibration mounts.

Energy is stored in a shock mount simply because it does deflect. To cause a shock mount to deflect requires a force. A force moving through a distance is basically the definition of work, and work can be defined in energy terms.

Consider the simple spring-mass system:
Kenetic energy, or the energy developed by a moving mass, is defined by:

$$KE = 1/2\ m\ V^2$$

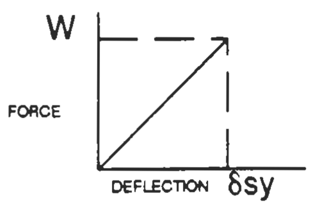


The potential energy of the spring-mass system is defined as:

$$PE = \int_0^{(\delta sy)_0} F_d \delta sy$$

where δsy is any deflection of the isolator.

Rather than perform the integration numerically, we will utilize a graphical approach. If the system is linear, its load-deflection curve will be as follows:



By graphical integration, the potential energy is equal to the area under the load-deflection curve (work done). Since the area under the curve is a triangle, the PE = 1/2 δW.

Recall that by the definition of spring rate,

$K = W/\delta$
solving for W: $W = K\delta$

therefore $PE = 1/2\, K\delta^2$

We know from the laws of conservation of energy that the total energy in a system must remain constant. Therefore, the total kenetic energy must equal the total potential energy.

$$1/2\, m V^2 = 1/2\, K\delta^2 \qquad (1)$$

solving for δ $$\delta = \sqrt{\frac{mv^2}{k}} \qquad (2)$$

solving for K $$K = \frac{mv^2}{\delta^2} \qquad (3)$$

The above analysis illustrates the technique of energy methods as a shock isolation approach. The value of the system Kenetic Energy can be determined readily if the velocity shock is known.

The velocity shock resulting from a free fall drop with inelastic impact (i.e., no rebound) is defined by: $Vi = \sqrt{2gh}$ and for an elastic impact (full rebound) $Ve = 2\sqrt{2gh}$ (where g is the acceleration of gravity and h is the drop height).

If the shock excitation is of the impulse type, the velocity changes are defined by:

half-sine:	$V = 2gAoto/\pi$	(4)	A_o = Acceleration
rectangular:	$V = gAoto$	(5)	T_o = time
triangular:	$V = gAoto/2$	(6)	g = gravity = 386 in/sec
versed sine:	$V = gAoto/2$	(7)	

Under impulsive shock excitation, the instantaneous velocity change V defined by the preceding data gives rise to sinusoidal acceleration and relative displacement responses having maximum magnitudes given by:

$$A\ max = \frac{fnV}{61.4} \qquad (8)$$

$$\delta\ max = \frac{V}{2\ \pi\ fn} \qquad (9)$$

These equations indicate that the transmitted acceleration is reduced by employing low values of the natural frequency, fn; however, use of a low natural frequency results in a larger relative deflection for a given velocity change. Consequently, a compromise between the maximum allowable acceleration and relative displacement response may be required in the selection of the isolation system natural frequency.

When dealing with a shock isolation problem, one must realize that the isolation system response is governed by the natural frequency of that system. Once the natural frequency is fixed, the response is fixed for any given input. Also the maximum response given by equation (8) is predicated upon the premise that the isolator, in fact, can deflect freely for the required deflection [equation (9)].

If the isolator cannot deflect freely and it "snubs' or if it is highly nonlinear, the transmitted acceleration will be much greater than equation (8) indicates. The effect of snubbing is exactly why most vibration isolators are not shock isolators but amplifiers. Shock and vibration isolation cannot be accomplished independently of each other. Often a compromise must be utilized; however, seemingly incompatible shock and vibration requirements can be the judicious application of nonlinear isolator characteristics.

Up to this point, shock has been discussed as only a velocity shock input. However, the shape of a shock impulse is influential in isolator design and isolation. In evaluating the effect of shock pulse shape on the dynamic response of linear systems, the concept of shock spectra is usually employed. Basically, a shock spectrum is a description of the manner in which the response maxima of single-degree-of-freedom systems vary with natural frequency and damping for a given shock excitation. Shaped pulses have been investigated in considerable detail with reference to shock spectra, and such data is readily available.

RANDOM VIBRATION - A BRIEF DISCUSSION

A new aspect of engineering concern came with man's entry into the era of high performance jets and missiles. This concern was for the failure of equipment packages which had been protected from sinusoidal vibration inputs. Even though the mountings used were effective in previous airborne applications, they proved ineffectual for this new environment; unexpectedly high excursion levels were experienced and the resultant occasional snubbing proved damaging to the mounted packages. The source of these excessive inputs was found in the area of the propulsion systems. These jet and rocket engines generated intense noise and vibration energy levels, random over a wide frequency range with resulting vibrations which were also random. It is the purpose of this paper to present a brief insight into the nature of this type of vibration phenomenon and to propose simple means of handing the design parameters associated with random vibration.

Random vibration is a dynamic situation which instantaneous values cannot be predicted as a time function. By virtue of its very nature, random vibration inputs (excursions) occur arbitrarily. For this reason, the characteristics of random vibration are described in statistical terms, specifically, by its power spectral density and the probability distribution of its magnitude. The first calculated parameter we use to simplify our analysis is the RMS (root- mean-squared) acceleration, given by the equation:

$$\ddot{X}\ RMS = \sqrt{S_f\,(f_2 - f_1\ )}$$

The RMS displacement is given by the expression:

$$X_{RMS} \ = \ 5.66\ \ddot{X}\ RMS / \sqrt{(\,f_1{}^3 - f_2\ )}$$

(Note: This is an approximate relationship which assumes f_1 is small relative to f_2.)

A single degree of freedom system responds to random vibration by oscillating at its natural frequency. The same system will also respond to "White Noise" by vibrating at its natural frequency except that the response varies in a random manner. The acceleration response magnitude is a function of the system natural frequency, resonant transmissibility, and input power spectral density. The equation for this is given by:

$$\ddot{Y}_{RMS} \text{ or } G_{RMS} = \sqrt{(\pi / 2)\ fn\ S_f Tr}$$

and its solution is found on the Barry slide rule. Your further work with random vibration will show this to be the single most important design relationship in the analysis.

The response displacement of the system is given the relationship:

$$Y_{RMS} = 9.8\ G_{RMS} / f_n^2$$

In applying a vibration isolator to meet a customer specification, the displacement requirement is usually the object of the analysis. Once the response displacement has been calculated, a factor of 3 (from probability theory) is applied. If the isolator has the required displacement available, then there is only a 0.3%* chance of exceeding the calculated excursion and thereby possibly snubbing. A simple calculation format is given by Barry Data Sheet No. 965.

The afore-mentioned relationships are based on an idealized simple system in order to simplify our analysis. When dealing with more complex multi-degree of freedom structures, one can express their solution as a series of single degree systems of known frequency and damping characteristics. Super-imposing the response of these simple systems will approximate the behavior of the more complex structure.

* - Based on Gaussian (normal) probability distribution

Our discussion has been intentionally brief in order to permit several problem solving exercises. In the course of your interaction with customer engineers, you will find that many random vibration questions are within your grasp.

Example 1: A customer has an airborne electronic package which is mounted on isolators displaying a fn of 30 Hz and maximum transmissibility of 3.0. The input power spectral density is 0.06 G^2 /Hz in the area of mount resonance. What displacement capability should the mounts have?

Solution: With given information:

$fn = 30$ Hz
$Tr = 3.0$
$Sf = 0.06\ G^2/Hz$

Calculate G_{RMS}: $\sqrt{(\pi/2)(3.0(30)(0.06)} = 2.91\ G_{RMS}$

Next calculate the RMS displacement (Y_{RMS})

$Y_{RMS} = (9.8)\ (2.91)/(30)^2 = 0.032$ inches

Multiple by probability factor of 3
(3) (.032) = .096 inches

Example 2: A customer wants to mount an equipment package aboard an aircraft. He wants to use your recommendation of Barry S-mounts but wants to make sure he does not exceed his fragility level of 10 G's due to the random input of 0.03 G^2/Hz from 0.5 to 100 Hz.

Solution: $fn = 7$ Hz & $Tr = 4$ (from Barry Catalog)
$Sf = 0.03\ G^2/Hz$ Fragility Level = 10 g's

From given information calculate G_{RMS}:
$G_{RMS} = 1.15$ G's

Probability theory tells us that there is only a 0.3% chance of exceeding a 3 G_{RMS} acceleration level. A 3 G_{RMS} level is 3.45 G's as compared with the fragility level of 20 G's so the design is considered quite safe. However, further calculation shows that the response displacement is:

$9.8\ G_{RMS}/fn^2 = 9.8(1.15)/49 = 0.23$ inches

and that a deflection capability of 3 times that or 0.69 inches is required. Since the S-mount does not have this deflection capability an alternative mounting must be sought. In all probability, a special mounting will be required for this application.

Terminology

f = frequency (Hz)

f_1 = lower limit of a frequency band (Hz)

f_2 = upper limit of a frequency band (Hz)

f_n = natural frequency (Hz)

G = gravitational constant (in/sec^2)

S_f = input power spectral density (G^2/Hz)

Tr = resonant transmissibility

X = instantaneous input displacement (inches)

X_{RMS} = root-mean-square input displacement (inches)

$\ddot{X}$ = instantaneous inputs acceleration (g's)

$\ddot{X}_{RMS}$ = root-mean-square input acceleration (g's)

Y = response displacement (inches)

$\ddot{Y}_{RMS} = G_{RMS}$ = root-mean-square response acceleration (g's)

Y_o = peak response displacement (inches)

$\ddot{Y}_o$ = peak response acceleration (g's)

Y_{RMS} = RMS response displacement (inches)

DEFINITIONS

Random Vibration - Random vibration is an oscillation having instantaneous magnitudes that vary in an upredictable manner and, therefore are not specified at any given instant of time. The characteristics of random vibration are described in statistical terms; specifically, random vibration is described by its spectral density and the probability distribution of its magnitude. Wide-band random vibration is comprised of a continuous spectrum of frequencies, whereas narrow-band random vibration has essentially a single frequency component and is often referred to as random sinusoidal vibration.

Stationary Vibration - Stationary vibration is that type of vibration for which properties, such as the mean magnitude, the rms magnitude, the spectral density, and the probability distribution of the random vibration magnitude, are independent of time. The condition of stationarity for periodic vibration.

White Noise - White Noise is a type of random vibration for which the spectral density has a constant value for all frequencies from zero to infinity.

Band-Limited White Noise - Band-limited white noise is a type of random vibration for which the spectral density has a constant value over a specified frequency range.

Power Density -The power density W(f) of random vibration is the mean-square magnitude per unit bandwidth of the output of an ideal filter with unity gain responding to the vibration, as follows:

$$W(f) = F(f)^2_{RMS}/\Delta f$$

where by convention the bandwidth f is usually chosen to be 1 Hz.

Power Density Spectrum - A power density spectrum is a graphical presentation of values of power density W(f) displayed as a function of frequency. It represents the distribution of vibration energy with frequency.

Normal Probability Distribution - The standardized form of the normal (or Gaussian) probability density, assuming a zero mean magnitude, is given by:

$$P(F/\sigma) = \left(1/\sqrt{2\pi}\right) e^{-1/2\,(F/\sigma)^2}$$

where σ is the standard deviation or rms magnitude of the variable F and

$$-\infty < F < \infty$$

The normal probability distribution has been found to describe suitably the statistical distribution of the instantaneous magnitude of random vibration.

RANDOM VIBRATION CAPABILITIES OF SELECTED MOUNTS

The tabulation which follows is given as an indicator of the input spectral density limits of selected standard product isolator series. The calculated limited are based on the following relationship:

$$G_{RMS} = \frac{(Y/3)fn^2}{9.8} \; ; \; Sf = G_{RMS}^2 \; / \; (\pi/2) \; fn \; Tr$$

It should be further noted that the limiting criteria is one of displacement capability within the mount and thereby its ability snubbing under "3 Y_{RMS}" excursion peaks.

MOUNT TYPE	fn (Hz)	Tr	MOUNT CLEARANCE INCHES S.A.	Sf (G^2/Hz)
990 & 915 Series	10	8	0.30	0.01
1000 Series Cupmount See note 1	25	8-N.R. 4.5-Univ. 3.5-Sil	0.23	0.08-NR 0.14-Univ. 0.17-Sil
2000 Series Cupmount See Note 1	25	8-N.R. 4.5-Univ. 3.5-Sil	0.23	0.08-NR 0.14-Univ. 0.17-Sil
4000 Series Cupmount See note 1	25	8-N.R. 4.5-Univ. 3.5-Sil	0.30	0.13-NR 0.23-Univ. 0.30-Sil
3000 Series Cupmount See Note 1	25	8-N.R. 4.5-Univ. 3.5-Sil	0.30	0.13-NR 0.23-Univ. 0.30-Sil
T-22	25	3.6	0.20	0.13
T-44	25	3.6	0.18	0.10
T-64	25	3.6	0.20	0.13
T-94	25	3.6	0.20	0.13
SLM Series	4	8	0.50	See note 2
B21 & 22	25	3.0	0.20	0.15
B43 & B44	25	3.0	0.20	0.15
B64	25	3.0	0.20	0.15

MOUNT TYPE	fn (Hz)	Tr	MOUNT CLEARANCE INCHES S.A.	Sf (G^2/Hz)
L21 & L22	8	1.5	0.20	See note 2
L-44	8	1.5	0.20	See note 2
L-64	8	1.5	0.15	See note 2
H-44	8	1.5	0.20	See note 2
H-64	8	1.5	0.20	See note 2
S-22	7	3.5	0.15	See note 2
S-44	7	3.5	0.15	See note 2
S-64	7	3.5	0.15	See note 2

Note: 1 Based on deflection capability before snubbing of lightest load bearing mount in each series.

2. Aircraft random vibration specifications usually initiate at a frequency above the resonant frequency for this isolator. For such situations, the random vibration environment is not a design concern.